Induction Task

$(L, L_B \subseteq L, L_E \subseteq L, L_H \subseteq L, \rightarrow)$

Given *background knowledge* $B \in L_B$, *positive examples* $E^+ \in L_E$ and *negative examples* $E^- \in L_E$, find a *hypothesis* $H \in L_H$, such that:

1. $B \not\rightarrow E^+$ (*necessity*);
2. $B \not\rightarrow E^-$ (*weak consistency*);
3. $B \cup H \rightarrow E^+$ (*sufficiency*);
4. $B \cup H \not\rightarrow E^-$ (*strong consistency*).

Hypothesis *generality/specificity*:
$H \geq H' \Leftrightarrow \{e | e \in L_E, H \rightarrow e\} \supseteq \{e | e \in L_E, H' \rightarrow e\}$.

Most general hypothesis $\top$: $e \in L_E \Rightarrow \top \rightarrow e$.

Most specific hypothesis $\bot$: $e \in E^+ \Leftrightarrow \bot \rightarrow e$.

Hypothesis space: $\{H | H \geq \bot, H \leq \top\}$

Generalization (specialization) *operators*:
$H' = \rho(H)$, $H' \geq H$ $(H' \leq H)$

Examples: gropping condition $(H_1 \geq H_2 \Leftrightarrow H_1 \subseteq H_2)$, term instantiation $(t_1 \geq t_2 \Leftrightarrow \exists \theta, t_1\theta = t_2)$.

Least general generalization (lgg): $H = lgg(H_1, H_2)$ iff $H \geq H_1$, $H \geq H_2$ and $\forall H'$, $H' \geq H_1$, $H' \geq H_2 \Rightarrow H' \geq H$.

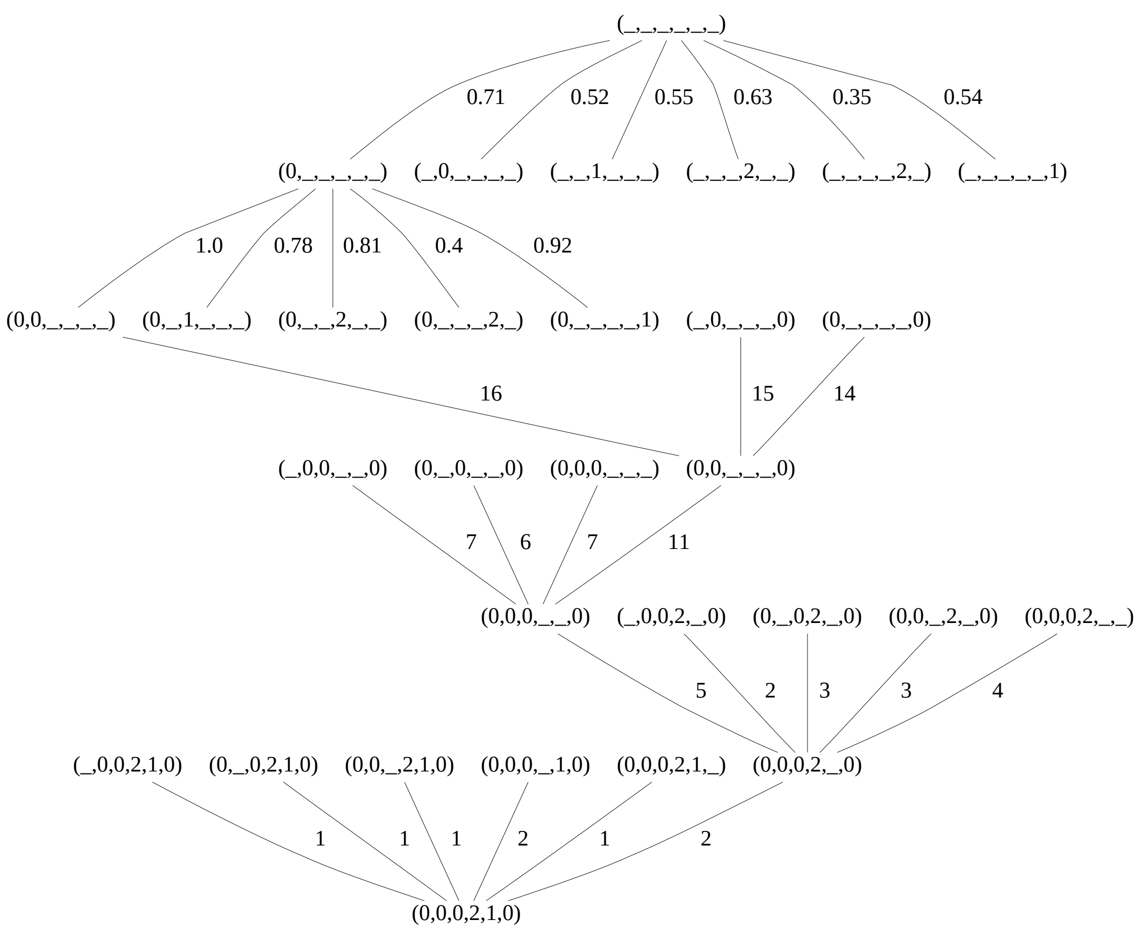
Examples: $lgg(H_1, H_2) = H_1 \cap H_2$, most specific *antiunifier* ($lgg(p(a,a), p(b,b)) = p(X,X)$).

Searching the hypothesis space – covering strategy

$H = H_1 \vee H_2 \vee ... \vee H_n$

1. find H_i, $\forall e \in E^- H_i \not\rightarrow e$;
2. $E^+ = E^+ \setminus \{e | H_i \rightarrow e\}$;
3. if $E^+ \neq \emptyset$ then go to 1 else stop.

Generalization/specialization graph



Lgg-based bottom-up search

Propositional language:
$E = \cup_{i=1}^{k} E_i$, $B = \emptyset$, $H = \{H_1, H_2, ..., H_k\}$, H_i satisfies the induction task for $E^+ = E_i$, $E^- = E \backslash E_i$, $B = \emptyset$.

1. choose $e_i, e_j \in E_k$ for some k;
2. $H_{ij}^k = lgg(e_i, e_j)$;
3. if $h_{ij}^k \not\to e, e \in E^l, l \neq k$ then
   $E^k = (E^K \setminus \{e | h_{ij} \to e\}) \cup \{h_{ij}\}$;
4. if step 1 or 3 are impossible then stop else go to 1.

Relational language:

θ-Subsumption: $C_1 \geq_\theta C_2 \Leftrightarrow \exists \theta \; C_1 \theta \subseteq C_2$.

$lgg_\theta(C_1, C_2) = \{L | L = lgg(L_1, L_2), L_i \in C_i, i = 1, 2\}$

Relative lgg:
$C = rlgg(C_1, C_2, B) \Leftrightarrow B \cup C \models C_1, B \cup C \models C_2$

$E = E^+ \cup E^-$, B – ground atoms, H – Horn clauses.

$rlgg(e_1, e_2, B) = lgg_\theta((e_1 : -B), (e_2 : -B))$

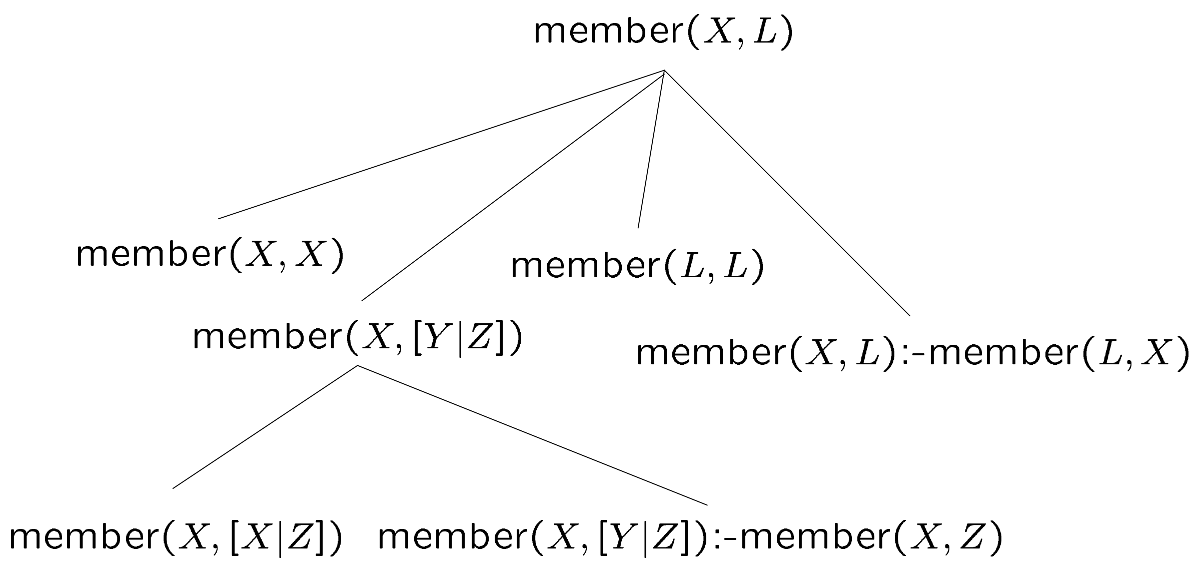
1. choose $e_i, e_j \in E^+$;
2. $C_{ij} = rlgg(e_i, e_j, B)$;
3. Reduce C_{ij}
4. $E^+ = E^+ \setminus \{e | B \cup C_{ij} \models e\}$;
5. if $E^+ = \emptyset$ then stop else go to 1.

Top-down relational learning

$E^+ = \{$member(a,[a,b]), member(b,[b]), member(b,[a,b])$\}$,
$E^- = \{$member(x,[a,b])$\}$

Specialization operatior (θ-subsumption):

- $\rho(C) = C\theta,\ |\theta| = 1$
- $\rho(C) = C \cup \neg L$



Leaves:

- successful: $C \models e \Rightarrow e \in E^+$
- unssuccesul: $\nexists e \in E^+, C \models e$.