Introduction, Course Overview and Number Systems

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Fall 2007

CS 113: Introduction to Computers

Lecture 1, September 4, 2007

Components of Computing Systems

- Hardware: Circuit boards, chips, disk drives, peripherals, wires, etc.
- Software: Programs (sequences of instructions for the computer to carry out)
- Data (information in its digital form)

Layers of a Computing System

Information

Hardware

Operating systems

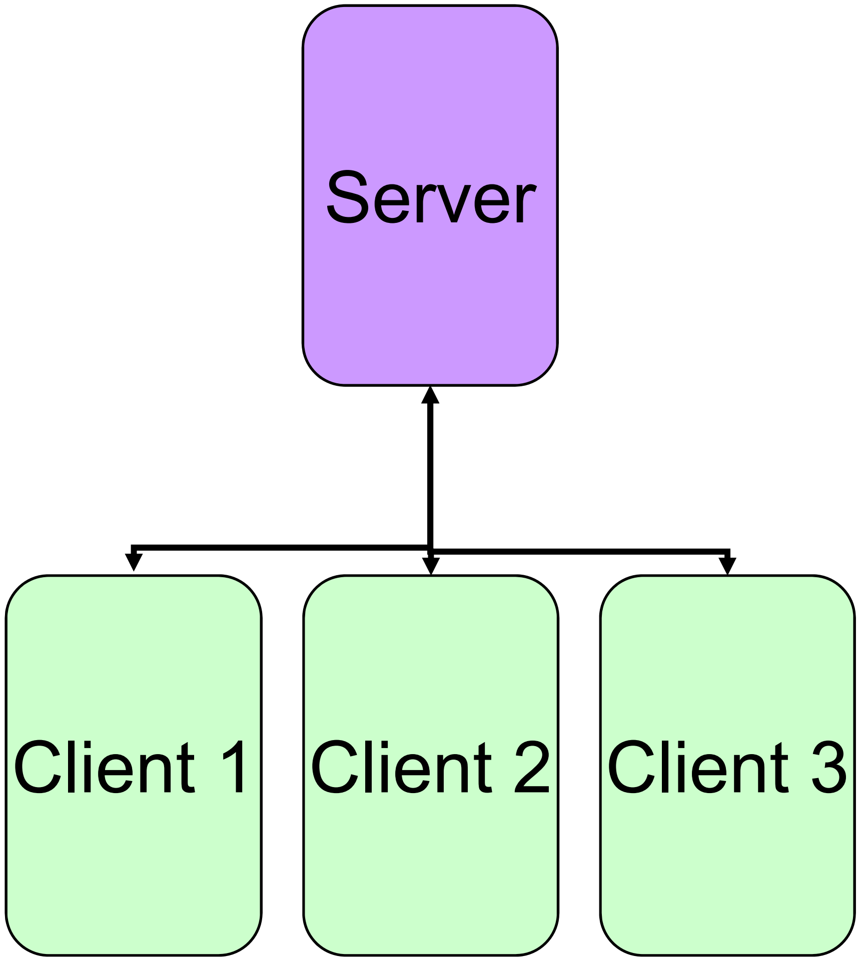
Applications

Communications

Programming:
- systems programming
- applications programming

Abstraction…

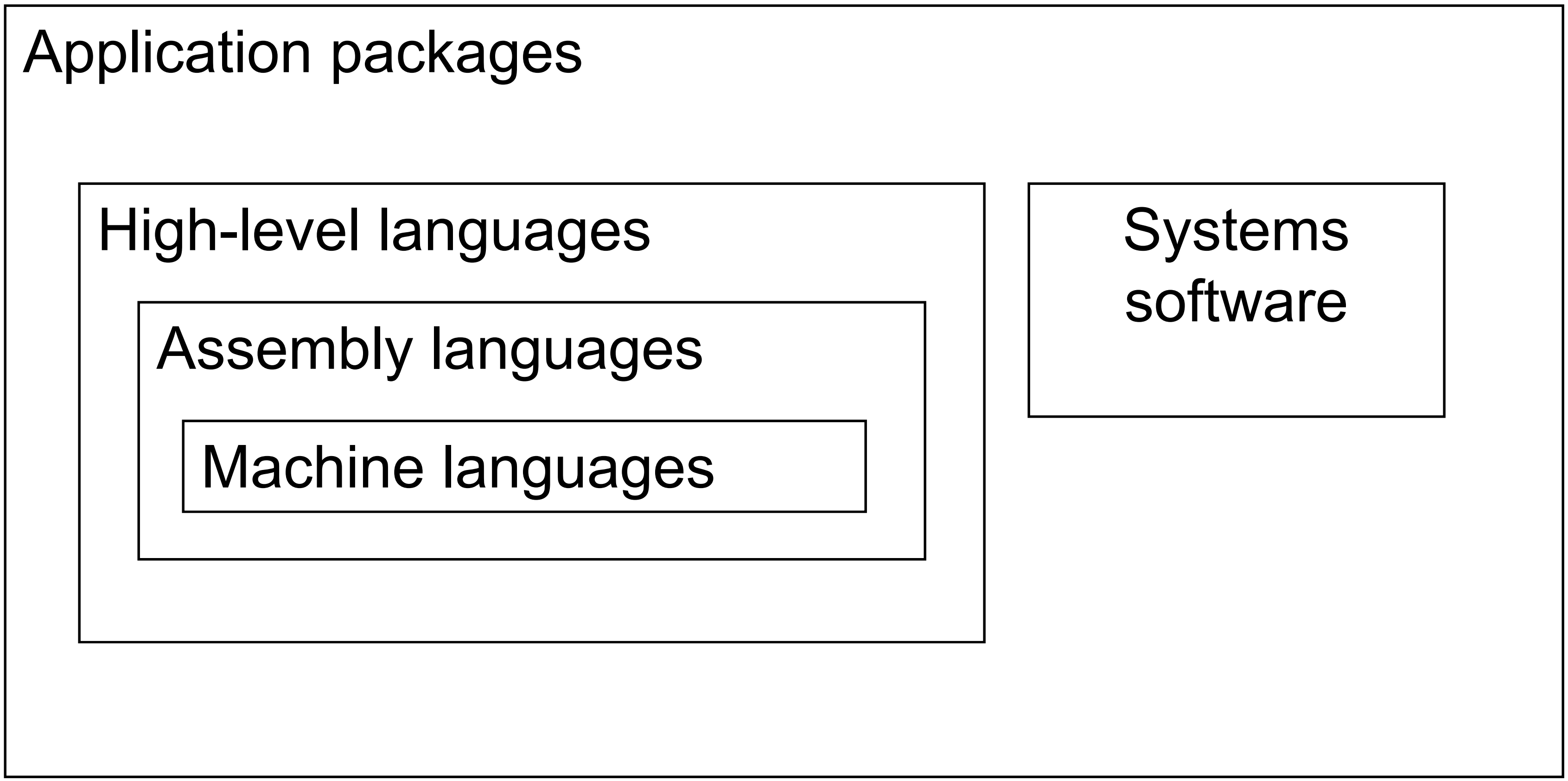
…removes or hides complex details.



The History of Computing

- Textbook, Section 1.5
- http://www.computerhistory.org/exhibits/internet_history/

Layers of Software



Number Categories

- *Natural numbers*: The number *0* and numbers obtained by repeatedly adding *1* to this number. Example: *3=0+1+1+1*
- *Negative numbers*: Less than *0*. Example: $-\sqrt{2}$
- *Integers*: Natural numbers and their negatives
- *Rational numbers*: Fractions, quotients of two integers. Examples: *16/13; 4/1=4*
- *Irrational numbers*: Cannot be represented as quotients of two integers. Example: $\sqrt{2}$

How to represent a natural number?

Base of a number system: The number of digits used in the system. Example 1: Base 10 (*decimal*)

$$1760_{10} = 0*10^0 + 6*10^1 + 7*10^2 + 1*10^3$$

Numbers are written using *positional notation*.

Example 2: Base 2 (*binary*)

$$11101_2 = 1*2^0 + 0*2^1 + 1*2^2 + 1*2^3 + 1*2^4 = 29_{10}$$

More Number Systems!

Example 3: *Octal* (Base 8)

$$73_8 = 3*8^0 + 7*8^1 = 59_{10} = 111011_2$$

Example 4: *Hexadecimal* (Base 16)

$$AF_{16} = 15*16^0 + 10*16^1 = 175_{10} = 257_8 = 10101111_2$$

Extra digits: A=10, B=11, C=12, D=13, E=14, F=15

How to represent a ratio?

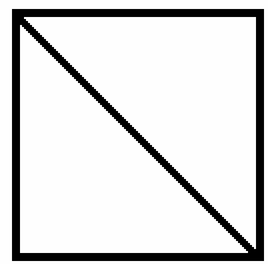
259:160=1.61875 241:149≈1.61745

A ratio is represented by an angle here.

How to represent an irrational number?

1. Geometrically

$$\sqrt{2}$$



2. By an algorithm: The Fibonacci numbers algorithm is a way to represent the golden ratio

$$\phi = \frac{1 + \sqrt{5}}{2} \approx 1.618034$$

Addition and Subtraction in Binary

1 11 ←carry			1 ←carry
10011	→	1+2+16 =	19
+ 11001	→	1+8+16 =	+ 25
101100	→	4+8+32 =	44

1 1 ←borrow		
10101	1+4+16 =	21
- 1011	1+2+ 8 =	- 11
1010	2+8 =	10

Power of Two Number Systems

1 digit in Base $8=2^3$ (octal) corresponds to

3 digits in Base 2 (binary):

$0_8 = 000_2$

$1_8 = 001_2$

$2_8 = 010_2$

$3_8 = 011_2$

$4_8 = 100_2$

$5_8 = 101_2$

$6_8 = 110_2$

$7_8 = 111_2$

Example of conversion:

$11001110_2 = (011)(001)(110)=316_8$

Indeed, $2+4+8+64+128=206_{10}$ and

$6+1*8+3*8^2 = 206_{10}$

Power of Two Number Systems (cont'd)

1 digit in Base $16=2^4$ (hexadecimal) corresponds to 4 digits in Base 2 (binary):

$0_{16} = 0000_2$	$8_{16} = 1000_2$
$1_{16} = 0001_2$	$9_{16} = 1001_2$
$2_{16} = 0010_2$	$A_{16} = 1010_2$
$3_{16} = 0011_2$	$B_{16} = 1011_2$
$4_{16} = 0100_2$	$C_{16} = 1100_2$
$5_{16} = 0101_2$	$D_{16} = 1101_2$
$6_{16} = 0110_2$	$E_{16} = 1110_2$
$7_{16} = 0111_2$	$F_{16} = 1111_2$

Example of conversion:

$11001110_2 = CE_{16}$

Converting from Base 10 to Other Bases

Converting 2849_{10} to hexadecimal (Base 16):

$2849/16=178.0625$; $178.0625-178=0.0625$;

$0.0625*16=1$, so 1 is the first digit from the right.

$178/16=11.125$; $11.125-11=0.125$;

$0.125*16=2$, so 2 is the second digit.

$11<16$, so B is the third and the last digit.

Indeed,

$B21_{16} = 1+2*16+11*16^2 = 2849_{10}$